

GENERATIVE AND COMPUTATIONAL POWER OF COMBINATORY CATEGORIAL GRAMMAR

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COMBINATORY CATEGORIAL GRAMMAR

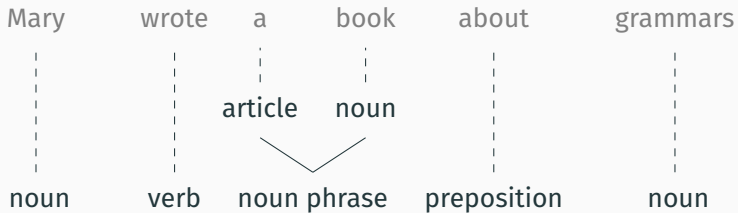
CONSTITUENTS IN NATURAL LANGUAGE

Mary wrote a book about grammars

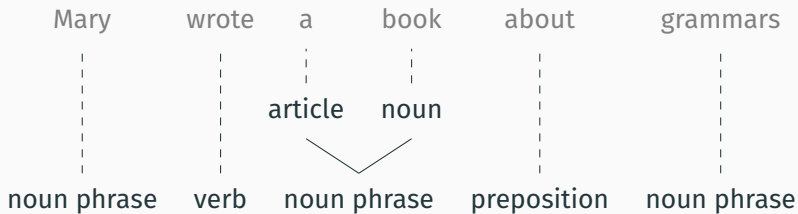
CONSTITUENTS IN NATURAL LANGUAGE

Mary	wrote	a	book	about	grammars
⋮	⋮	⋮	⋮	⋮	⋮
noun	verb	article	noun	preposition	noun

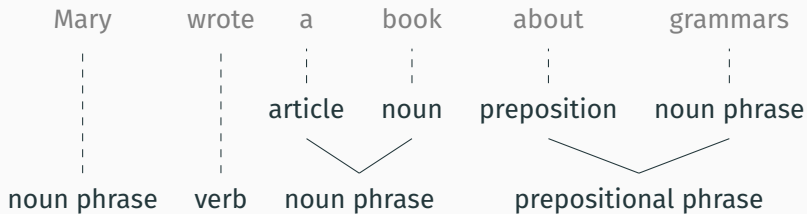
CONSTITUENTS IN NATURAL LANGUAGE



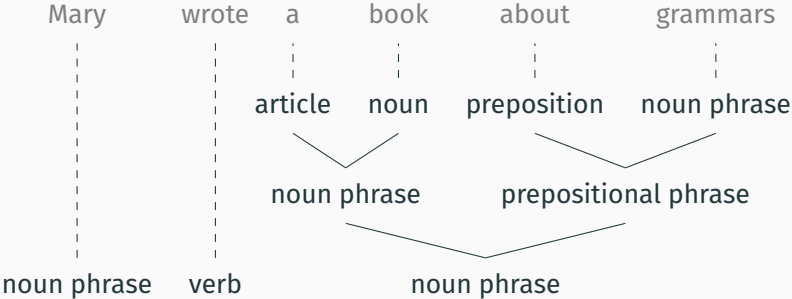
CONSTITUENTS IN NATURAL LANGUAGE



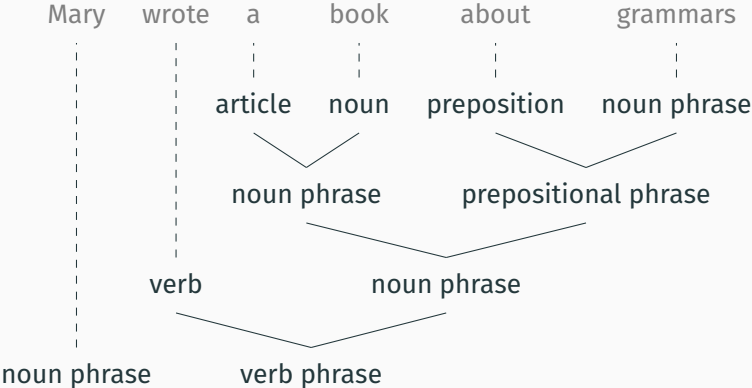
CONSTITUENTS IN NATURAL LANGUAGE



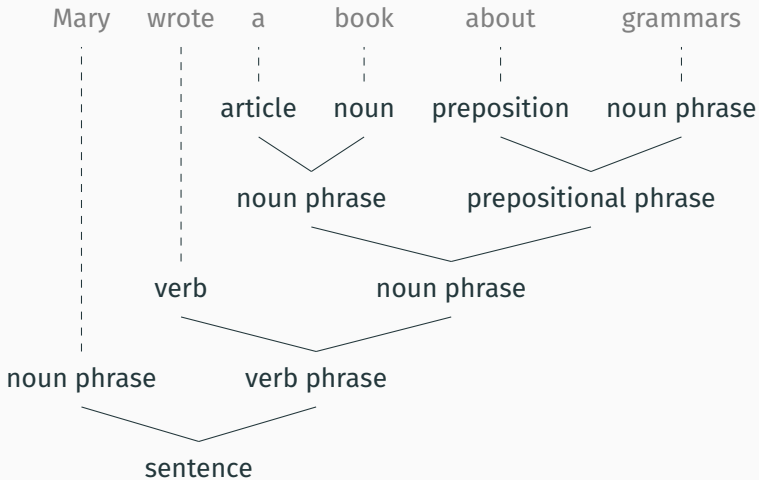
CONSTITUENTS IN NATURAL LANGUAGE



CONSTITUENTS IN NATURAL LANGUAGE

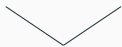


CONSTITUENTS IN NATURAL LANGUAGE



CATEGORY REPRESENTATION OF CONSTITUENTS

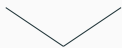
article noun



noun phrase

CATEGORY REPRESENTATION OF CONSTITUENTS

article noun



noun phrase

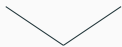
NP/N N



NP

CATEGORY REPRESENTATION OF CONSTITUENTS

article noun



noun phrase

NP/N N



NP

article category NP/N

“obtain a noun phrase if a noun is on the right side”

CATEGORY REPRESENTATION OF CONSTITUENTS

article noun

noun phrase

NP/N N

NP

article category NP/N

$f: N \rightarrow NP$

“obtain a noun phrase if a noun is on the right side”

CATEGORY REPRESENTATION OF CONSTITUENTS

article noun

noun phrase

NP/N N

NP

article category NP/N

$f: N \rightarrow NP$

“obtain a noun phrase if a noun is on the right side”

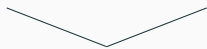
NP, N atoms

NP target

/N argument

CATEGORY REPRESENTATION OF CONSTITUENTS

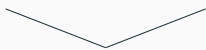
noun phrase verb phrase



sentence

CATEGORY REPRESENTATION OF CONSTITUENTS

noun phrase verb phrase



sentence

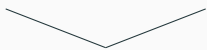
NP S\NP



S

CATEGORY REPRESENTATION OF CONSTITUENTS

noun phrase verb phrase



sentence

NP S\NP



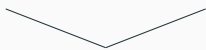
S

verb phrase category **S\NP**

“obtain a **sentence** if a **noun phrase** is on the **left side**”

CATEGORY REPRESENTATION OF CONSTITUENTS

noun phrase verb phrase



sentence

NP S\NP



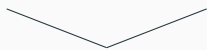
S

verb phrase category $S \setminus NP$ $f: NP \rightarrow S$

“obtain a **sentence** if a **noun phrase** is on the **left side**”

CATEGORY REPRESENTATION OF CONSTITUENTS

noun phrase verb phrase



sentence

NP S\NP



S

verb phrase category **S\NP** $f: NP \rightarrow S$

“obtain a **sentence** if a **noun phrase** is on the **left side**”

S, NP atoms

S target

\NP argument

CATEGORY REPRESENTATION OF CONSTITUENTS

transitive verb category (example: *likes*)

S\NP/NP

CATEGORY REPRESENTATION OF CONSTITUENTS

transitive verb category (example: *likes*)

S\NP/NP

“obtain a **sentence** if there is

- a **noun phrase** on the **right side** and
- a **noun phrase** on the **left side**”

CATEGORY REPRESENTATION OF CONSTITUENTS

transitive verb category (example: *likes*)

$S \backslash NP / NP$ $f: NP \times NP \rightarrow S$

“obtain a **sentence** if there is

- a **noun phrase** on the **right side** and
- a **noun phrase** on the **left side**”

CATEGORY REPRESENTATION OF CONSTITUENTS

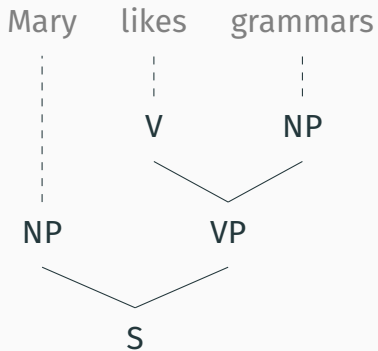
transitive verb category (example: *likes*)

S \ **NP** / **NP** $f: NP \times NP \rightarrow S$ or $f: NP \rightarrow (NP \rightarrow S)$

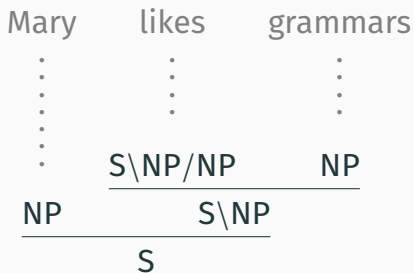
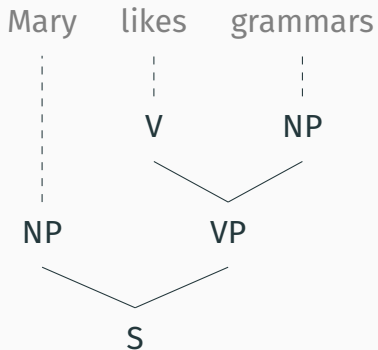
“obtain a **sentence** if there is

- a **noun phrase** on the **right side** and
- a **noun phrase** on the **left side**”

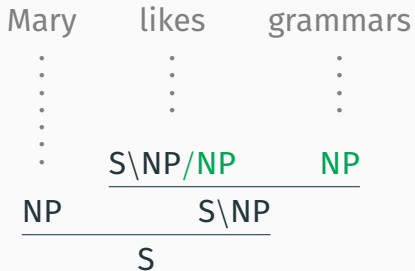
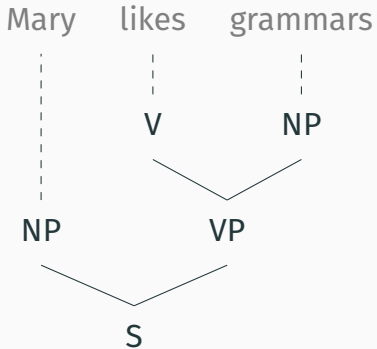
CONSTITUENCY TREE VS CCG DERIVATION TREE



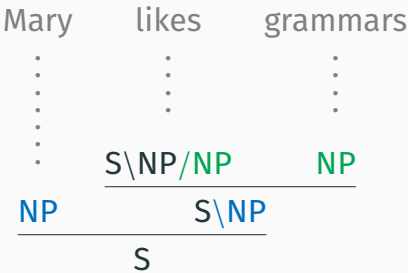
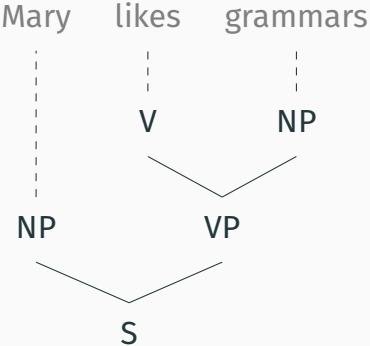
CONSTITUENCY TREE VS CCG DERIVATION TREE



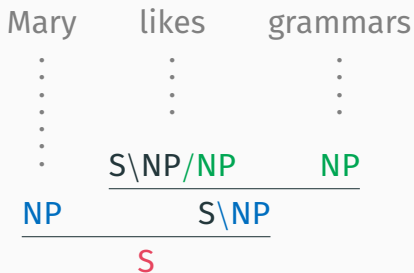
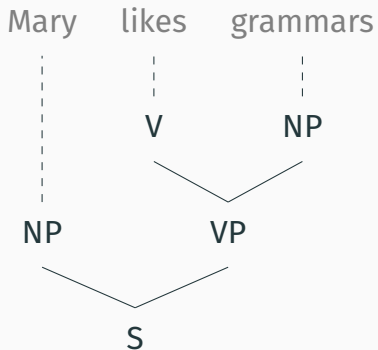
CONSTITUENCY TREE VS CCG DERIVATION TREE



CONSTITUENCY TREE VS CCG DERIVATION TREE



CONSTITUENCY TREE VS CCG DERIVATION TREE



S initial category

APPLICATION RULES

forward application

$$\frac{c/b \quad b}{c}$$

backward application

$$\frac{b \quad c \backslash b}{c}$$

c/b , $c \backslash b$ primary category
 b secondary category
 c output category

APPLICATION RULES

forward application

$$\frac{c/b \quad b}{c}$$

backward application

$$\frac{b \quad c/b}{c}$$

c/b , c/b primary category
 b secondary category
 c output category


$$\frac{S \backslash NP / NP \quad NP}{S \backslash NP}$$

COMPOSITION RULES

forward rule

$$\frac{c/b \quad b\beta}{c\beta}$$

backward rule

$$\frac{b\beta \quad c \setminus b}{c\beta}$$

β argument sequence
 $|\beta|$ rule degree

COMPOSITION RULES

degree 0

$$\frac{c/b \quad b}{c}$$

COMPOSITION RULES

degree 0

$$\frac{c/b \quad b}{c}$$

degree 1

$$\frac{c/b \quad b \backslash e}{c \backslash e}$$

COMPOSITION RULES

degree 0

$$\frac{c/b \quad b}{c}$$

degree 1

$$\frac{c/b \quad b \backslash e}{c \backslash e}$$

$$f: e \rightarrow b \quad g: b \rightarrow c$$

$$f \circ g: e \rightarrow c$$

COMPOSITION RULES

degree 0

$$\frac{c/b \quad b}{c}$$

degree 1

$$\frac{c/b \quad b \backslash e}{c \backslash e}$$

degree 2

$$\frac{c \backslash b \quad b \backslash e / d}{c \backslash e / d}$$

$$f: e \rightarrow b \quad g: b \rightarrow c$$

$$f \circ g: e \rightarrow c$$

COMPOSITION RULES

degree 0

$$\frac{c/b \quad b}{c}$$

degree 1

$$\frac{c/b \quad b \backslash e}{c \backslash e}$$

degree 2

$$\frac{c \backslash b \quad b \backslash e / d}{c \backslash e / d}$$

$$f: e \rightarrow b \quad g: b \rightarrow c$$

$$f \circ g: e \rightarrow c$$

In a rule we may restrict

- the **secondary category** to a concrete category
- the **target** of the **primary category** to a concrete atom

$$\frac{c/b \quad b}{c} \rightarrow \frac{S_X/NP \quad NP}{S_X}$$

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

- input alphabet $\Sigma = \{\alpha, \beta\}$
- atomic categories $A = \{D, E\}$
- initial categories $I = \{D\}$
- lexicon L with
 - $L(\alpha) = \{D/E, D/E/D\}$
 - $L(\beta) = \{E\}$
- rule set includes
all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

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α α β β

- rule set includes
all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

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all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

α	α	β	β
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		\cdot	\cdot
		E	E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

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$$L(\alpha) = \{D/E, D/E/D\}$$

$$L(\beta) = \{E\}$$

- rule set includes
all application rules

$$R = \left\{ \frac{c/b \quad b \quad b \quad c \setminus b}{c}, \frac{\quad}{c} \right\}$$

α	α	β	β
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot
$D/E/D$	D/E	E	E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

- input alphabet $\Sigma = \{\alpha, \beta\}$
- atomic categories $A = \{D, E\}$
- initial categories $I = \{D\}$

- lexicon L with

$$L(\alpha) = \{D/E, D/E/D\}$$

$$L(\beta) = \{E\}$$

- rule set includes
all application rules

$$R = \left\{ \frac{c/b \quad b \quad b \quad c \setminus b}{c}, \frac{c \setminus b}{c} \right\}$$

α	α	β	β
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
$D/E/D$	D/E	E	E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

- input alphabet $\Sigma = \{\alpha, \beta\}$
- atomic categories $A = \{D, E\}$
- initial categories $I = \{D\}$

- lexicon L with

$$L(\alpha) = \{D/E, D/E/D\}$$

$$L(\beta) = \{E\}$$

- rule set includes
all application rules

$$R = \left\{ \frac{c/b \quad b \quad b \quad c \setminus b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

α	α	β	β
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	D/E	E	⋮
$D/E/D$	$\frac{D/E \quad E}{D}$	E	E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

- input alphabet $\Sigma = \{\alpha, \beta\}$
- atomic categories $A = \{D, E\}$
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$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

α	α	β	β
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
$D/E/D$	$\frac{D/E}{E}$	E	E
$D/E/D$	D	E	E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

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all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

α	α	β	β
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	D/E	E	⋮
$D/E/D$	D		⋮
D/E			E

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

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 - $L(\alpha) = \{D/E, D/E/D\}$
 - $L(\beta) = \{E\}$
- rule set includes all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \backslash b}{c} \right\}$$

$$\begin{array}{cccc} \alpha & \alpha & \beta & \beta \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & D/E & E & \vdots \\ \vdots & \frac{D/E \quad E}{D} & \vdots & \vdots \\ \frac{D/E/D}{D/E} & \frac{D}{D} & \vdots & E \end{array}$$

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

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 - $L(\beta) = \{E\}$
- rule set includes all application rules

$$R = \left\{ \frac{c/b \quad b}{c}, \frac{b \quad c \setminus b}{c} \right\}$$

$$\begin{array}{cccc}
 \alpha & & \alpha & \beta & \beta \\
 \vdots & & \cdot & \cdot & \vdots \\
 \vdots & & & & \vdots \\
 \cdot & & D/E & E & \vdots \\
 \hline
 D/E/D & & D & & \vdots \\
 \hline
 D/E & & & & E \\
 \hline
 & & D & &
 \end{array}$$

EXAMPLE CCG $\mathcal{G} = (\Sigma, A, R, I, L)$

- input alphabet $\Sigma = \{\alpha, \beta\}$
- atomic categories $A = \{D, E\}$
- initial categories $I = \{D\}$
- lexicon L with
 - $L(\alpha) = \{D/E, D/E/D\}$
 - $L(\beta) = \{E\}$

- rule set includes
restricted rules

$$R = \left\{ \frac{Dx/D \ D}{Dx}, \frac{Dx/E \ E}{Dx} \right\}$$

$$\begin{array}{cccc}
 \alpha & \alpha & \beta & \beta \\
 \vdots & \cdot & \cdot & \vdots \\
 \vdots & & & \vdots \\
 \cdot & D/E & E & \vdots \\
 \hline
 D/E/D & D & & \vdots \\
 \hline
 D/E & & & E \\
 \hline
 & D & &
 \end{array}$$

CLASSICAL LANGUAGE CLASSES



CSG Context-Sensitive Grammar

LBA Linear Bounded Automaton

CFG Context-Free Grammar

PDA Push-Down Automaton

CROSS-SERIAL DEPENDENCIES

... omdat ik Cecilia Henk de nijlpaarden zag helpen voeren.
... because I Cecilia Henk the hippopotamuses saw help feed

The diagram consists of two rows of text. The top row is the Dutch sentence: "... omdat ik Cecilia Henk de nijlpaarden zag helpen voeren." The bottom row is the English sentence: "... because I Cecilia Henk the hippopotamuses saw help feed". Vertical lines connect the words in the English sentence to the words in the Dutch sentence: 'I' to 'ik', 'Cecilia' to 'Cecilia', 'Henk' to 'Henk', 'the' to 'de', 'hippopotamuses' to 'nijlpaarden', 'saw' to 'zag', 'help' to 'helpen', and 'feed' to 'voeren'. Horizontal lines connect the words in the English sentence to the words in the Dutch sentence: a line from 'I' to 'zag', a line from 'Cecilia' to 'helpen', a line from 'Henk' to 'voeren', a line from 'the' to 'voeren', a line from 'hippopotamuses' to 'zag', a line from 'saw' to 'voeren', a line from 'help' to 'zag', and a line from 'feed' to 'zag'. These horizontal lines cross each other, illustrating cross-serial dependencies.

'... because I saw Cecilia help Henk feed the hippopotamuses.'

Example from [Steedman \(1985\)](#)

CROSS-SERIAL DEPENDENCIES

... omdat ik Cecilia Henk de nijlpaarden zag helpen voeren.
... because I Cecilia Henk the hippopotamuses saw help feed

The diagram illustrates cross-serial dependencies between the Dutch sentence "... omdat ik Cecilia Henk de nijlpaarden zag helpen voeren." and the English sentence "... because I Cecilia Henk the hippopotamuses saw help feed". Vertical lines connect corresponding words: "ik" to "I", "Cecilia" to "Cecilia", "Henk" to "Henk", "de" to "the", "nijlpaarden" to "hippopotamuses", "zag" to "saw", "helpen" to "help", and "voeren" to "feed". Horizontal lines connect the start and end of phrases that are interleaved in the original order: from "I" to "saw", from "Cecilia" to "help", and from "Henk" to "feed".

'... because I saw Cecilia help Henk feed the hippopotamuses.'

$$\text{COPY} = \{ ww \mid w \in \Sigma^* \}$$

Example from [Steedman \(1985\)](#)

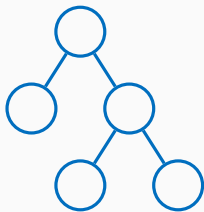
MILD CONTEXT-SENSITIVITY



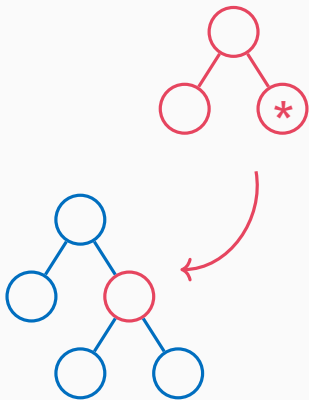
MCFG Multiple Context-Free Grammar

TAG Tree-Adjoining Grammar

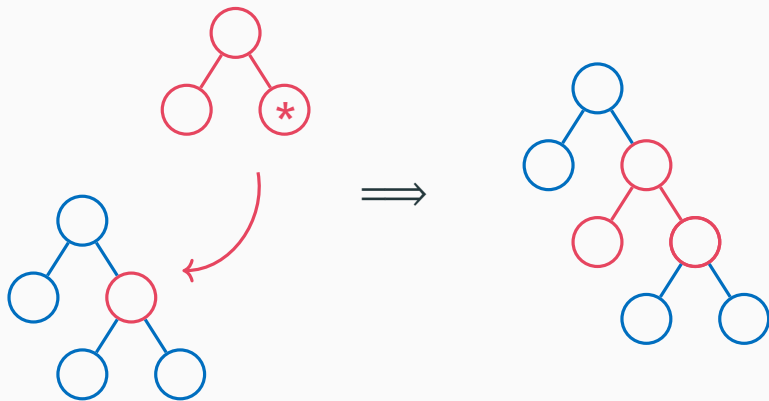
TREE-ADJOINING GRAMMAR



TREE-ADJOINING GRAMMAR



TREE-ADJOINING GRAMMAR



MILD CONTEXT-SENSITIVITY OF CCG

Math. Systems Theory 27, 511-546 (1994)

**Mathematical
Systems
Theory**
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The Equivalence of Four Extensions of Context-Free Grammars*

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² School of Cognitive and Computing Sciences, University of Sussex,
Brighton, Sussex BN1 9QH, England

Abstract. There is currently considerable interest among computational linguists in grammatical formalisms with highly restricted generative power. This paper concerns the relationship between the class of string languages generated by several such formalisms, namely, combinatory categorial grammars, head grammars, linear indexed grammars, and tree adjoining grammars. Each of these formalisms is known to generate a larger class of languages than context-free grammars. The four formalisms under consideration were developed independently and appear superficially to be quite different from one another. The result presented in this paper is that all four of the formalisms under consideration generate exactly the same class of string languages.

1. Introduction

There is currently considerable interest among computational linguists in grammatical formalisms with highly restricted generative power. This is based on the argument that a grammar formalism should not merely be viewed as a notation, but as part of the linguistic theory [6]. It should make predictions about the structure of natural language and its value is lessened to the extent that it supports both good and bad analyses. In order for a grammar formalism to have such predictive power its generative capacity must be constrained. This has led to

* This work has been supported by NSF Grants MCS-82-19116-CER, MCS-82-07294, DCR-84-10413, IRI-8909610, ARO Grant DAA29-84-9-0027, and DARPA Grant N0014-83-K0018.

TAG = CCG = LIG = HG

Vijay-Shanker, Weir (1994)

LIG Linear Indexed Grammar
HG Head Grammar

MILD CONTEXT-SENSITIVITY OF CCG

Parsing Some Constrained Grammar Formalisms

K. Vijay-Shanker^{*}
University of Delaware

David J. Weir[†]
University of Sussex

In this paper we present a scheme to extend a recognition algorithm for Context-Free Grammars (CFG) that can be used to derive polynomial-time recognition algorithms for a set of formalisms that generate a superset of languages generated by CFG. We describe the scheme by developing a Cocke-Kasami-Younger (CKY)-like pure bottom-up recognition algorithm for Linear Indexed Grammars and show how it can be adapted to give algorithms for Tree Adjoining Grammars and Combinatory Categorical Grammars. This is the only polynomial-time recognition algorithm for Combinatory Categorical Grammars that we are aware of.

The main contribution of this paper is the general scheme we propose for parsing a variety of formalisms whose derivation process is controlled by an explicit or implicit stack. The ideas presented here can be suitably modified for other parsing styles or used in the generalized framework set out by Lang (1990).

1. Introduction

This paper presents a scheme to extend known recognition algorithms for Context-Free Grammars (CFG) in order to obtain recognition algorithms for a class of grammatical formalisms that generate a strict superset of the set of languages generated by CFG. In particular, we use this scheme to give recognition algorithms for Linear Indexed Grammars (LIG), Tree Adjoining Grammars (TAG), and a version of Combinatory Categorical Grammars (CCG). These formalisms belong to the class of *mildly context-sensitive grammar formalisms* identified by Joshi (1985) on the basis of some properties of their generative capacity. The parsing strategy that we propose can be applied to the formalisms listed as well as others that have similar characteristics (as outlined below) in their derivational process. Some of the main ideas underlying our scheme have been influenced by the observations that can be made about the constructions used in the proofs of the equivalence of these formalisms and Head Grammars (HG) (Vijay-Shanker 1987; Weir 1988; Vijay-Shanker and Weir 1993).

There are similarities between the TAG and HG derivation processes and that of Context-Free Grammars (CFG). This is reflected in common features of the parsing algorithms for HG (Pollard 1984) and TAG (Vijay-Shanker and Joshi 1985) and the CKY algorithm for CFG (Kasami 1965; Younger 1967). In particular, what can happen at each step in a derivation can depend only on which of a finite set of "states" the derivation is in (for CFG, these states can be considered to be the nonterminal symbols). This property, which we refer to as the **context-freeness property**, is important because it allows one to keep only a limited amount of context during the recognition process.

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CCG parsable in $O(|w|^6)$

Vijay-Shanker, Weir (1994)

COMPUTATIONAL COMPLEXITY

PARSING DECISION PROBLEMS

Membership Problem

- input: w
- question: $w \in \mathcal{L}(\mathcal{G})$?

can be solved in $O(|w|^6)$ Vijay-Shanker, Weir (1994)

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can be solved in $O(|w|^6)$ Vijay-Shanker, Weir (1994)

Universal Recognition Problem

- input: w, \mathcal{G}
- question: $w \in \mathcal{L}(\mathcal{G})$?

EXPTIME-/NP-complete Kuhlmann, Satta, Jonsson (2018)

LEXICON ENTRIES FOR THE EMPTY WORD

ϵ -entries

lexicon entries	ϵ
for the	\vdots
empty word ϵ	$B/D/C$

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classical proof for [equivalence of TAG and CCG](#)
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	.
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universal recognition problem

- with ϵ -entries [EXPTIME-complete](#)
- without them [NP-complete](#)

[Kuhlmann, Satta, Jonsson \(2018\)](#)

RESEARCH QUESTION

CCG variant	complexity
with ε -entries	EXPTIME [†]
without ε -entries	NP [†]
?	PTIME

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CCG variant	complexity
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- Can we restrict CCG such that parsing becomes **polynomial in the grammar size**?
- Can we find a **practically relevant** formalism with this property?

[†]Kuhlmann, Satta, Jonsson (2018)

APPROACH

- inclusion of substitution rules

$$\frac{c/b/e \quad b/e}{c/e}$$

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→ generalized rule notation

$$\frac{c/b\alpha \quad b\alpha\beta}{c\alpha\beta} \qquad \frac{b\alpha\beta \quad c \setminus b\alpha}{c\alpha\beta} \qquad \text{with } |\alpha| \leq 1$$

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- new parsing algorithm based on [Kuhlmann, Satta \(2014\)](#)
- complexity in terms of grammar size:
new runtime **exponential** only in **maximum rule degree k**

PARSING AS DEDUCTION

Parsing is viewed as a **deductive process**:

- start from a set of **axioms** and derive new **items**

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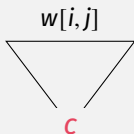
- input is accepted if **goal item** is derived

ITEM TYPES

Tree Items

$[c, i, j]$

represents a derivation tree
with root category c

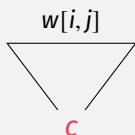


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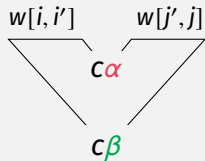


Context Items

$[\alpha, \beta, i, i', j', j]$

$1 \leq |\alpha| \leq 2$

$|\beta| \leq \text{maximum rule degree } k$



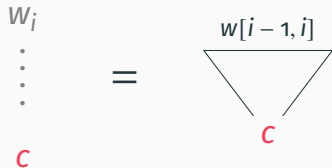
PARSING ALGORITHM – AXIOMS AND GOAL

AXIOMS: Lexicon Entry \rightarrow Tree

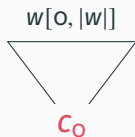
$$\begin{array}{c} w_i \\ \vdots \\ \vdots \\ \vdots \\ c \end{array} = \begin{array}{c} w[i-1, i] \\ \triangle \\ c \end{array}$$

PARSING ALGORITHM – AXIOMS AND GOAL

AXIOMS: Lexicon Entry \rightarrow Tree



GOAL: Tree over complete input with c_0 initial



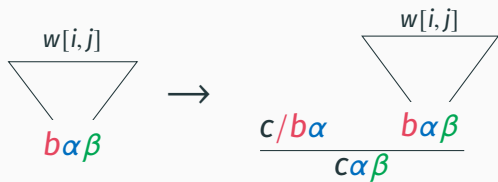
PARSING ALGORITHM – RULE 1

① Tree \rightarrow Context



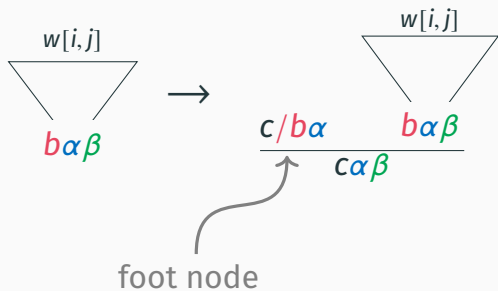
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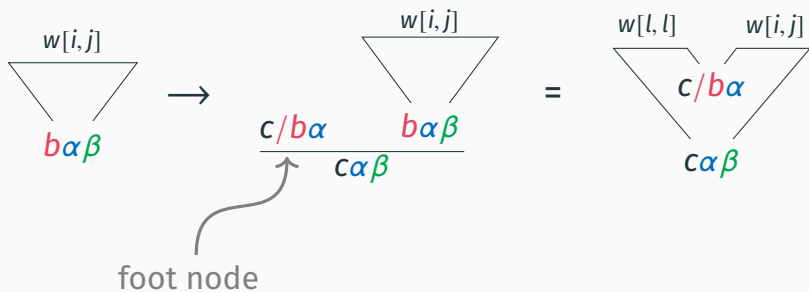
PARSING ALGORITHM – RULE 1

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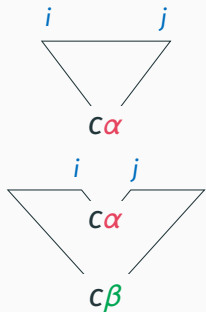
PARSING ALGORITHM – RULE 1

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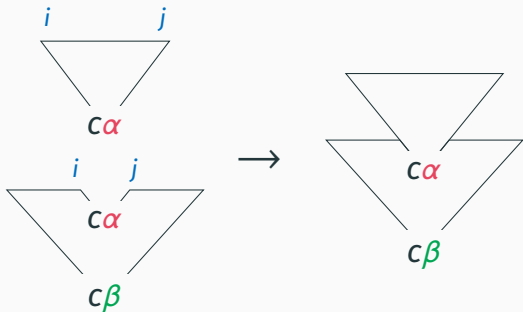
PARSING ALGORITHM – RULE 2

② Tree + Context \rightarrow Tree



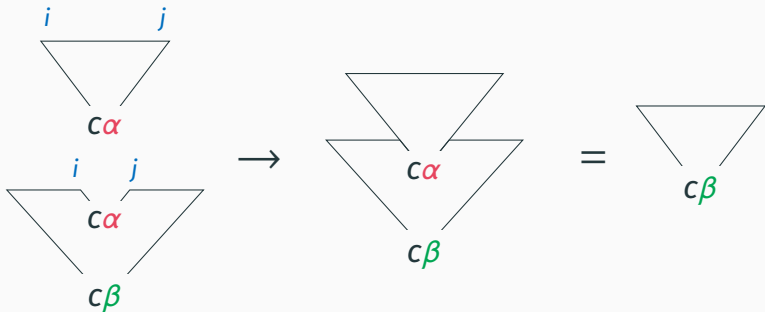
PARSING ALGORITHM – RULE 2

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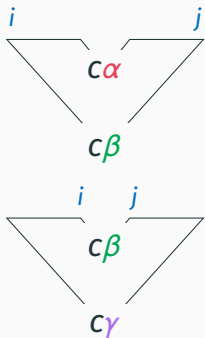
PARSING ALGORITHM – RULE 2

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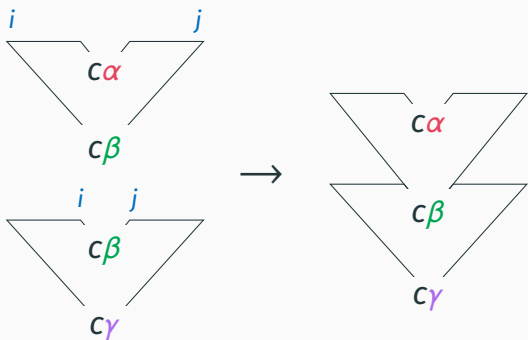
PARSING ALGORITHM – RULE 3

③ Context + Context \rightarrow Context



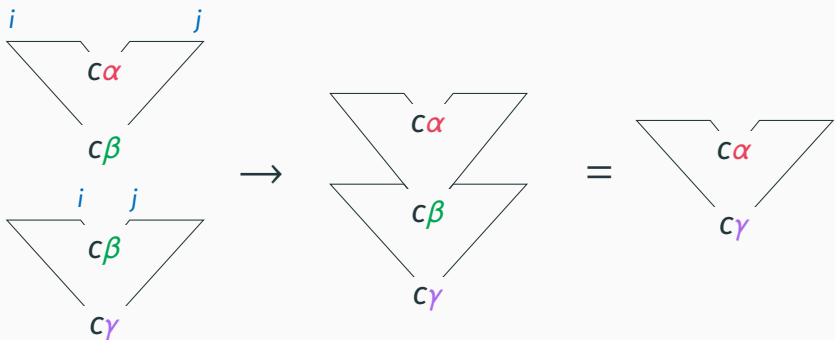
PARSING ALGORITHM – RULE 3

③ Context + Context \rightarrow Context



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WHAT IS NEW?

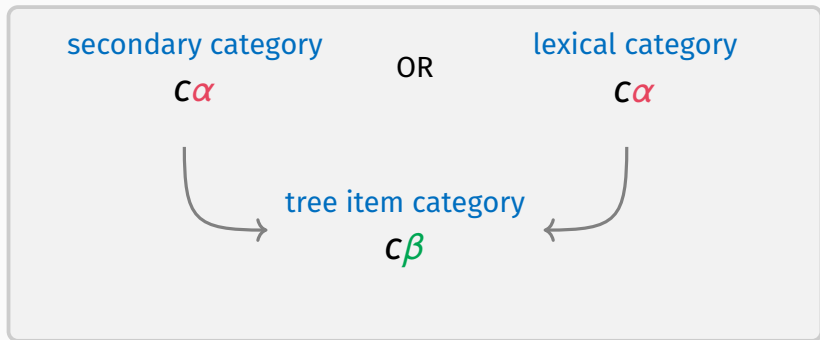
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WHAT IS NEW?

- generalization to **substitution rules**
- improve complexity by restricting the **tree items**

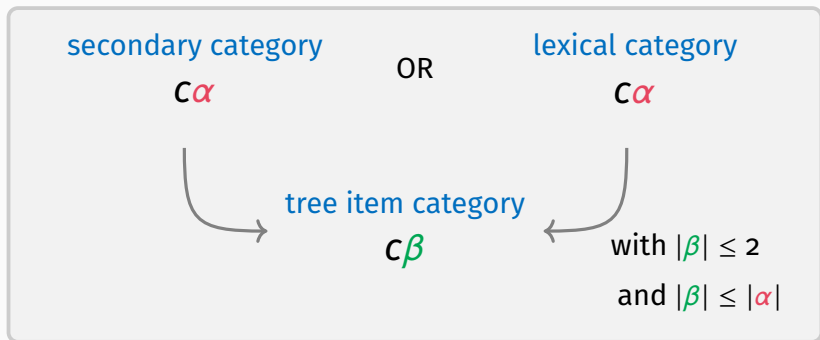
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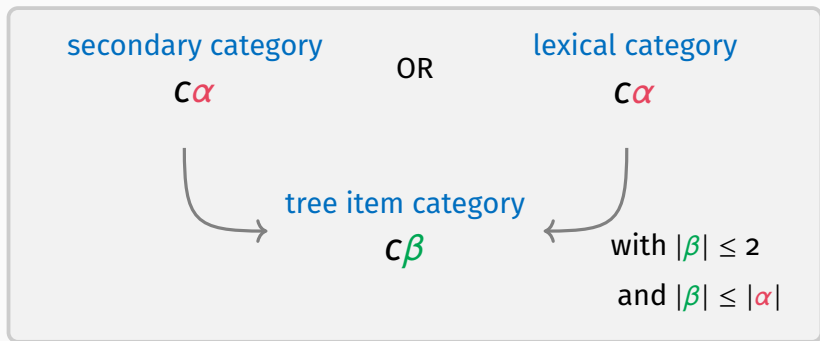
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WHAT IS NEW?

- generalization to **substitution rules**
- improve complexity by restricting the **tree items**



→ number of items (and deduction rules!) **exponential in k**

COMPLEXITY RESULT

Theorem

The universal recognition problem for k -CCG with ϵ -entries and substitution rules can be solved in time and space $O(|\mathcal{G}|^{k+5} \cdot |w|^6)$.

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The universal recognition problem for k -CCG with ε -entries and substitution rules can be solved in time and space $O(|\mathcal{G}|^{k+5} \cdot |w|^6)$.

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with ε -entries	EXPTIME [†]
without ε -entries	NP [†]
bounded degree	PTIME

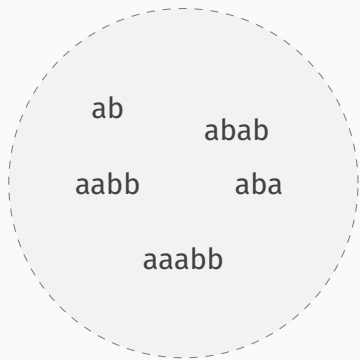
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GENERATIVE CAPACITY

STRONG GENERATIVE CAPACITY

weak

generative capacity



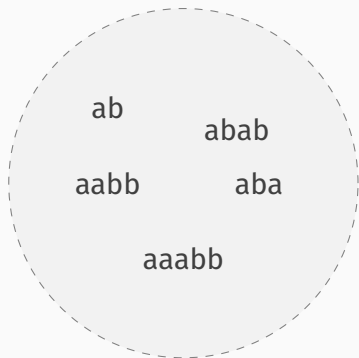
string languages

= sets of strings

STRONG GENERATIVE CAPACITY

weak

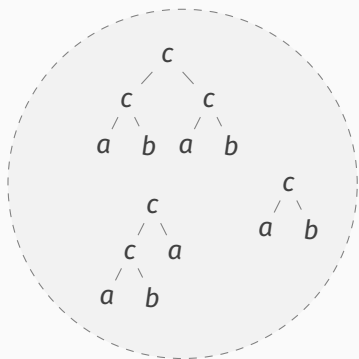
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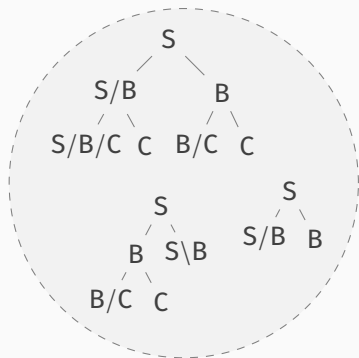
strong

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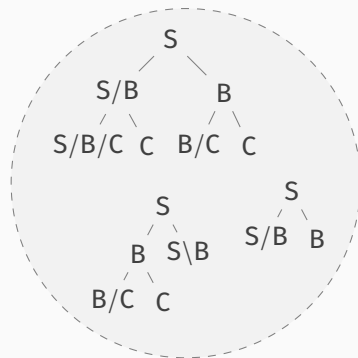
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TREE LANGUAGE OF CCG



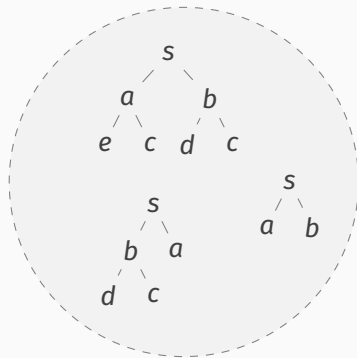
CCG derivation tree set
(root category = initial)

TREE LANGUAGE OF CCG



CCG derivation tree set
(root category = initial)

→
relabel



generated
tree language

OVERVIEW OF GENERATIVE CAPACITY

	0-CCG	1-CCG	2-CCG	k -CCG	k -CCG with ε -entries
strings	= CFG ^a	= CFG ^b			= TAG ^c
trees	\subseteq RTG ^d				

RTG Regular Tree Grammar

^aBar-Hillel, Gaifman, Shamir (1964)

^bFowler, Penn (2010)

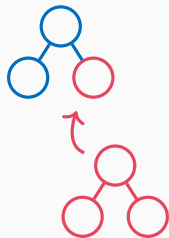
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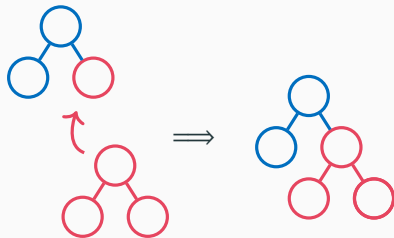
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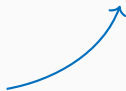
How does the **rule degree** affect the generative capacity?

STRONG EQUIVALENCE OF TAG AND CCG

TAG = [†]SCFTG

simple monadic

context-free tree grammar

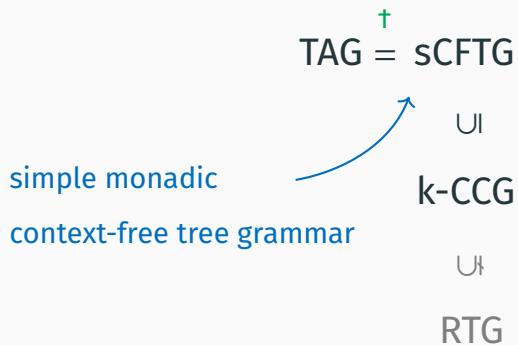


U†

RTG

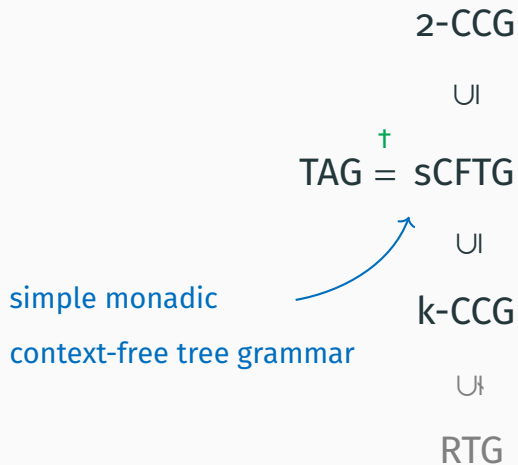
[†]Kepser, Rogers (2011)

STRONG EQUIVALENCE OF TAG AND CCG



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STRONG EQUIVALENCE OF TAG AND CCG

2-CCG

UI

TAG = [†]SCFTG = CCG


UI

k-CCG

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STRONG EQUIVALENCE OF TAG AND CCG

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UI

k-CCG

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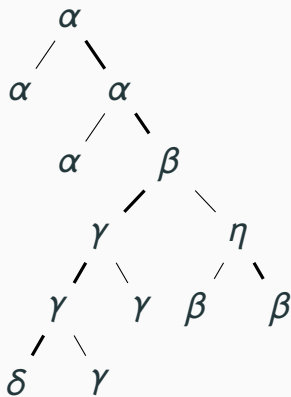
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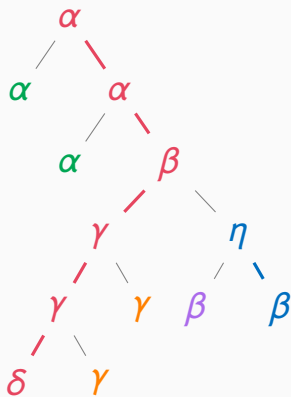


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DECOMPOSITION INTO SPINES



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DECOMPOSITION INTO SPINES

$$\delta - \gamma - \gamma - \beta - \alpha - \alpha$$

$$\beta - \eta$$

α

γ

β

\vdots

DECOMPOSITION INTO SPINES

$\delta - \gamma - \gamma - \beta - \alpha - \alpha$

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α

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\vdots

} context-free

DECOMPOSITION INTO SPINES

$\delta - \gamma - \gamma - \beta - \alpha - \alpha$

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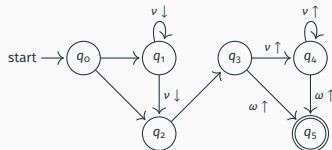
α

γ

β

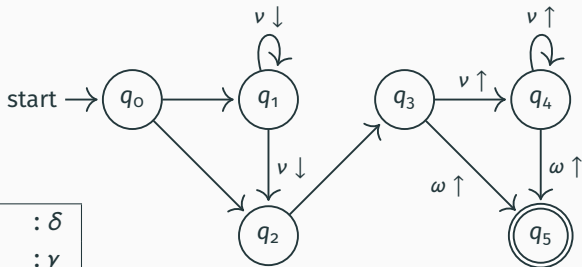
\vdots

context-free



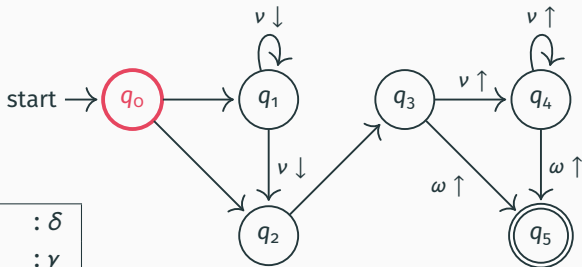
Moore PDA

MOORE PDA



q_0	: δ
q_1, q_2	: γ
q_3	: β
q_4, q_5	: α

MOORE PDA

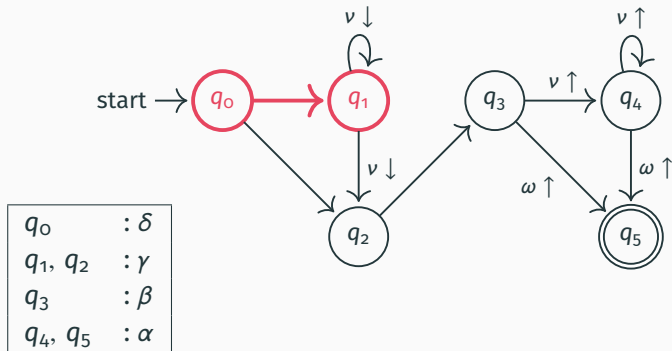


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$\langle q_0, \boxed{\omega} \rangle$

δ

MOORE PDA

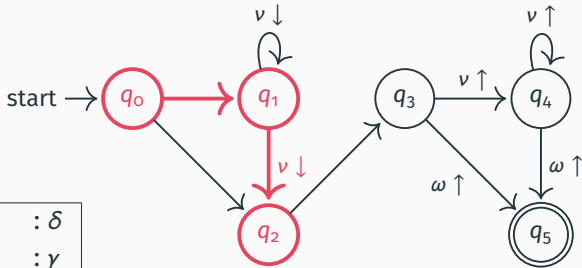


$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle$

δ

γ

MOORE PDA



q_0	: δ
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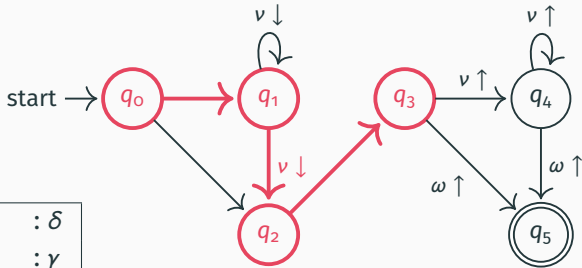
$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle$$

δ

γ

γ

MOORE PDA



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$\langle q_0, \omega \rangle \vdash \langle q_1, \omega \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle$

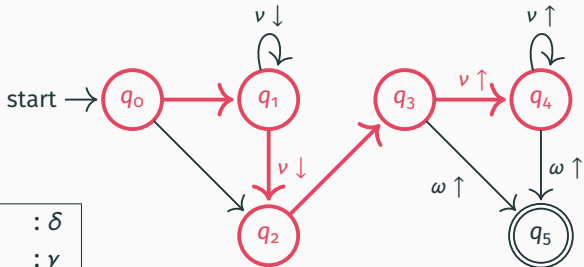
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δ

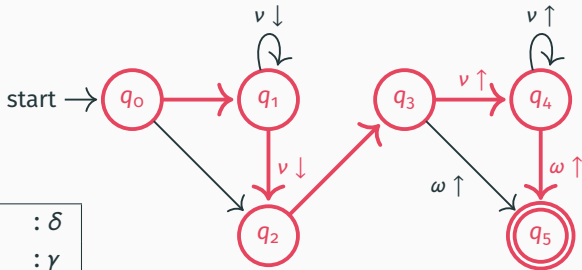
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δ

γ

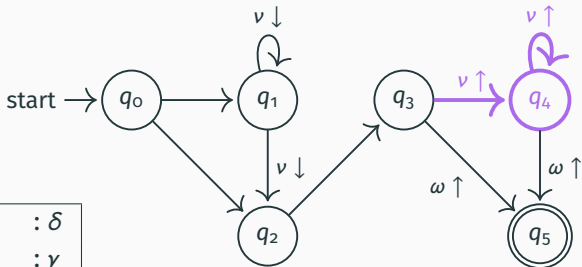
γ

β

α

α

MOORE PDA



q_0	: δ
q_1, q_2	: γ
q_3	: β
q_4, q_5	: α

$\langle q_0, \omega \rangle \vdash \langle q_1, \omega \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \omega \rangle \vdash \langle q_5, _ \rangle$

δ

γ

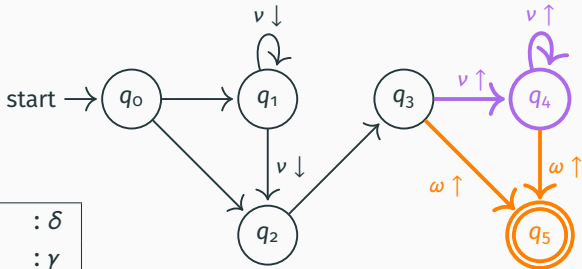
γ

β

α

α

MOORE PDA



q_0	: δ
q_1, q_2	: γ
q_3	: β
q_4, q_5	: α

$\text{pop}(v) = q_4$

$\text{pop}(\omega) = q_5$

$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$

δ

γ

γ

β

α

α

SIMULATE MOORE PDA

- Moore PDA generates all spines (of length ≥ 2)

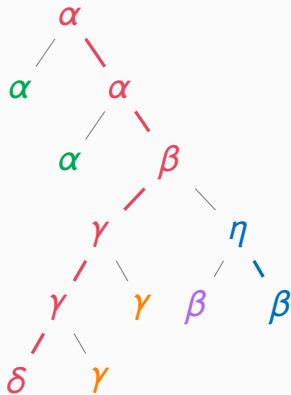
SIMULATE MOORE PDA

- Moore PDA generates all spines (of length ≥ 2)
- primary category length can grow unbounded
 - simulate Moore PDA in primary spines of CCG
 - store stack in the argument sequence

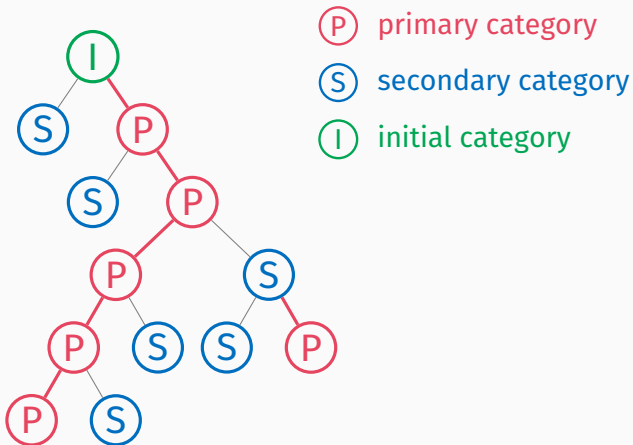
SIMULATE MOORE PDA

- Moore PDA generates all spines (of length ≥ 2)
- primary category length can grow unbounded
 - simulate Moore PDA in primary spines of CCG
 - store stack in the argument sequence
- last argument of primary category stores
 - current state
 - topmost stack symbol

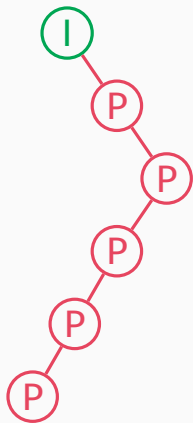
PRIMARY SPINES



PRIMARY SPINES

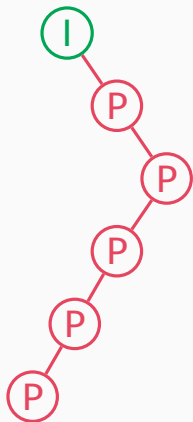


SIMULATE MOORE PDA



$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$

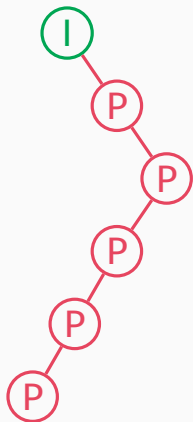
SIMULATE MOORE PDA



$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix}$$

$$\langle q_0, \omega \rangle \vdash \langle q_1, \omega \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \omega \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA

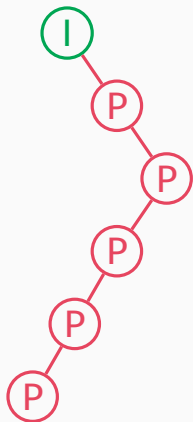


$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$

$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix}$

$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$

SIMULATE MOORE PDA



$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix}$$

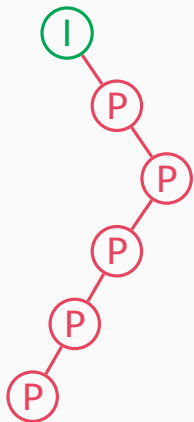
$$\text{pop}(v) = q_4$$

$$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix}$$

$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA



$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_3 \\ v \end{pmatrix}$$

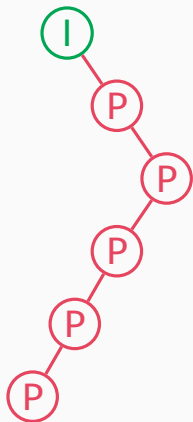
$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix} \quad \text{pop}(v) = q_4$$

$$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix}$$

$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA



$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_3 \\ v \end{pmatrix}$$

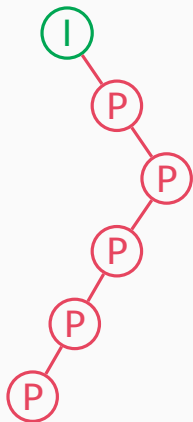
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$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA



c

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_3 \\ v \end{pmatrix}$$

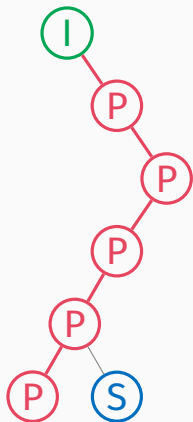
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SIMULATE MOORE PDA



c

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_3 \\ v \end{pmatrix}$$

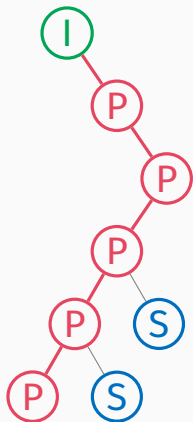
$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix} \quad \text{pop}(v) = q_4$$

$$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix} \quad \begin{pmatrix} q_0 \\ \omega \end{pmatrix} / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA



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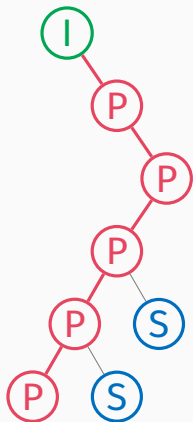
$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix} \quad \text{pop}(v) = q_4$$

$$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix} \quad \begin{pmatrix} q_1 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix}$$

$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix} \quad \begin{pmatrix} q_0 \\ \omega \end{pmatrix} / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, _ \rangle$$

SIMULATE MOORE PDA



$$c = \begin{pmatrix} \perp \\ \varepsilon \\ q_5 \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_3 \\ v \end{pmatrix}$$

$$c \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix} \quad \text{pop}(v) = q_4$$

$$c / \begin{pmatrix} q_1 \\ \omega \end{pmatrix} \quad \begin{pmatrix} q_1 \\ \omega \end{pmatrix} \setminus \begin{pmatrix} q_4 \\ \omega \end{pmatrix} / \begin{pmatrix} q_2 \\ v \end{pmatrix}$$

$$c / \begin{pmatrix} q_0 \\ \omega \end{pmatrix} \quad \begin{pmatrix} q_0 \\ \omega \end{pmatrix} / \begin{pmatrix} q_1 \\ \omega \end{pmatrix}$$

$$\langle q_0, \boxed{\omega} \rangle \vdash \langle q_1, \boxed{\omega} \rangle \vdash \langle q_2, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_3, \begin{array}{|c|} \hline v \\ \hline \omega \\ \hline \end{array} \rangle \vdash \langle q_4, \boxed{\omega} \rangle \vdash \langle q_5, \underline{\quad} \rangle$$

STRONG EQUIVALENCE RESULT

TAG = sCFTG

U

k-CCG

STRONG EQUIVALENCE RESULT

2-CCG

UI

TAG = sCFTG

UI

k-CCG

STRONG EQUIVALENCE RESULT

2-CCG

UI

TAG = sCFTG

UI

k-CCG with ε -entries

STRONG EQUIVALENCE RESULT

2-CCG without ε -entries

UI

TAG = sCFTG

UI

k-CCG with ε -entries

STRONG EQUIVALENCE RESULT

2-CCG without ε -entries

UI

TAG = sCFTG = 2-CCG without ε -entries

UI

k-CCG with ε -entries

STRONG EQUIVALENCE RESULT

2-CCG without ε -entries

UI

TAG = sCFTG = 2-CCG without ε -entries

UI

k-CCG with ε -entries

Theorem

2-CCG without ε -entries generates the same class of tree languages as TAG.

OVERVIEW OF GENERATIVE CAPACITY

	0-CCG	1-CCG	2-CCG	k -CCG	k -CCG with ε -entries
strings	= CFG ^a	= CFG ^b			= TAG ^c
trees	\subsetneq RTG ^d				

^aBar-Hillel, Gaifman, Shamir (1964)

^bFowler, Penn (2010)

^cVijay-Shanker, Weir (1994)

^dBuszkowski (1988)

OVERVIEW OF GENERATIVE CAPACITY

	0-CCG	1-CCG	2-CCG	k -CCG	k -CCG with ε -entries
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trees	\subsetneq RTG ^d	= RTG			

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OVERVIEW OF GENERATIVE CAPACITY

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^aBar-Hillel, Gaifman, Shamir (1964)

^bFowler, Penn (2010)

^cVijay-Shanker, Weir (1994)

^dBuszkowski (1988)

OVERVIEW OF RESULTS






CCG variant	rule degree	ε -entries	string languages	tree languages
(pure) with application rules only	$k = 0$	yes/no	= CFG	\subsetneq RTG
pure with composition	$k = 1$	yes/no	= CFG	\subsetneq RTG
composition	$k = 1$	yes/no	= CFG	= RTG
pure with composition	$k \geq 2$	yes/no	\subsetneq TAG	\subsetneq TAG
prefix-closed, no target restrictions	$k \geq 2$	yes/no	\subsetneq TAG	\subsetneq TAG
prefix-closed	$k \geq 2$	yes	= TAG	
composition	$k \geq 2$	no	= TAG	= TAG
composition	$k \geq 2$	yes	= TAG	= TAG
composition and substitution	$k \geq 2$	yes	= TAG	
generalized composition	unlimited	no	\supsetneq TAG	\supsetneq TAG
generalized composition	unlimited	yes	\supsetneq TAG	\supsetneq TAG

OVERVIEW OF RESULTS

CCG variant	rule degree	ε -entries	string languages	tree languages
(pure) with application rules only	$k = 0$	yes/no	= CFG	\subsetneq RTG
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composition	$k = 1$	yes/no	= CFG	= RTG
pure with composition	$k \geq 2$	yes/no	\subsetneq TAG	\subsetneq TAG
prefix-closed, no target restrictions	$k \geq 2$	yes/no	\subsetneq TAG	\subsetneq TAG
prefix-closed	$k \geq 2$	yes	= TAG	
composition	$k \geq 2$	no	= TAG	= TAG
composition	$k \geq 2$	yes	= TAG	= TAG
composition and substitution	$k \geq 2$	yes	= TAG	
generalized composition	unlimited	no	\supsetneq TAG	\supsetneq TAG
generalized composition	unlimited	yes	\supsetneq TAG	\supsetneq TAG

CCG variant	complexity
with ε -entries	EXPTIME
without ε -entries	NP
bounded degree	PTIME

PUBLICATIONS

-  Marco Kuhlmann, Andreas Maletti, and Lena K. Schiffer. **THE TREE-GENERATIVE CAPACITY OF COMBINATORY CATEGORIAL GRAMMARS.** *FSTTCS*, 2019.
-  Marco Kuhlmann, Andreas Maletti, and Lena K. Schiffer. **THE TREE-GENERATIVE CAPACITY OF COMBINATORY CATEGORIAL GRAMMARS.** *Journal of Computer and System Sciences*, 2022.
-  Lena K. Schiffer and Andreas Maletti. **STRONG EQUIVALENCE OF TAG AND CCG.** *Transactions of the Association for Computational Linguistics*, 2021.
-  Lena K. Schiffer, Marco Kuhlmann, and Giorgio Satta. **TRACTABLE PARSING FOR CCGS OF BOUNDED DEGREE.** *Computational Linguistics*, 2022.
-  Andreas Maletti and Lena K. Schiffer. **COMBINATORY CATEGORIAL GRAMMARS AS GENERATORS OF WEIGHTED FORESTS.** *Information and Computation*, 2023.